

# Viewing Graph Solvability in Structure from Motion

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# Outline

- Introduction
- Calibrated Case
- Uncalibrated Case
- Calibrated vs Uncalibrated
- Conclusion

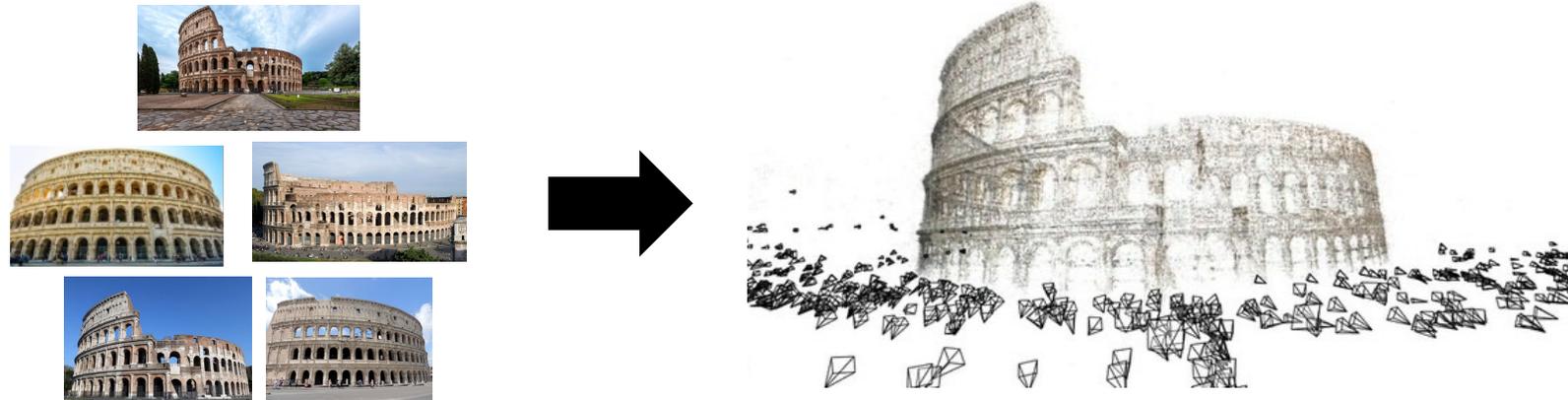
# Outline

- **Introduction**
- Calibrated Case
- Uncalibrated Case
- Calibrated vs Uncalibrated
- Conclusion

# Introduction

The goal of **structure from motion** (SfM) is to recover both camera motion and scene structure, starting from point correspondences in multiple images:

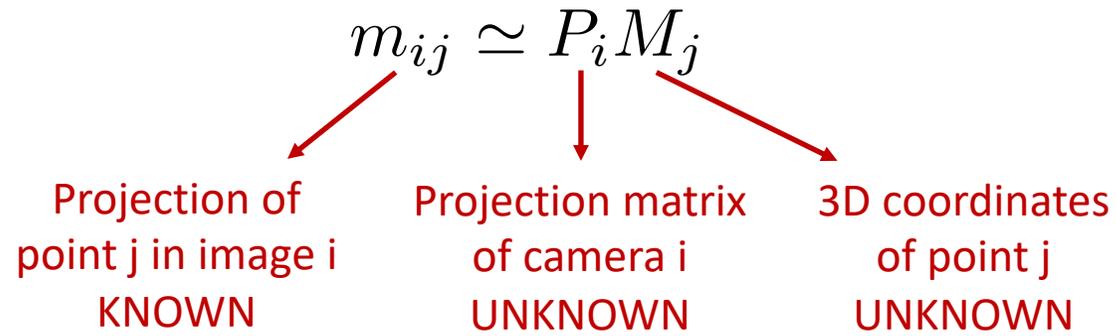
- camera motion = camera matrices/poses;
- scene structure = 3D coordinates of points.



■ O. Ozyesil, V. Voroninski, R. Basri, A. Singer. *A survey of structure from motion*. Acta Numerica (2017).

# Introduction

Formally, the task is to compute **camera matrices**  $P_i$  and **coordinates of 3D points**  $M_j$  starting from image points  $m_{ij}$  such that the following equation is best satisfied:

$$m_{ij} \simeq P_i M_j$$


Projection of point j in image i  
KNOWN

Projection matrix of camera i  
UNKNOWN

3D coordinates of point j  
UNKNOWN

In the calibrated case, calibration matrices are known and projection matrices consist of **rotations** and **translations**:

$$P_i = K_i [R_i \quad \mathbf{t}_i]$$


Known    Unknown

# Introduction

Is 3D reconstruction *unique*?



The solution is defined (at least) up to a global **projective transformation**:

$$m_{ij} \simeq P_i M_j = P_i \underbrace{Q Q^{-1}}_{\text{identity}} M_j = \underbrace{P_i Q}_{\text{new cameras}} \underbrace{Q^{-1} M_j}_{\text{new points}}$$

If cameras are calibrated, then the reconstruction ambiguity is represented (at least) by a global **rotation, translation and scale**.

# Introduction

The task of solvability is to analyse the **ambiguities** inherent to the SfM problem:

- single transformation → well-posed problem ✓
- multiple transformations → ill-posed problem ✗

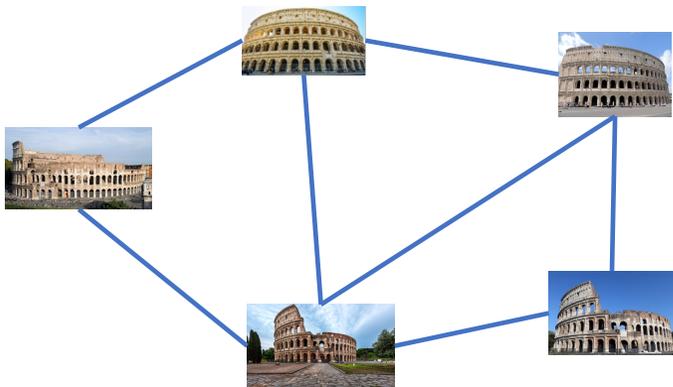
*There are many ways to approach SfM!* ⚠

Here we focus on a framework that recovers **camera motion** from two-view relationships only (no points):

- Essential matrix (calibrated)
- Fundamental matrix (uncalibrated)

# Introduction

The problem can be represented as a **viewing graph**:

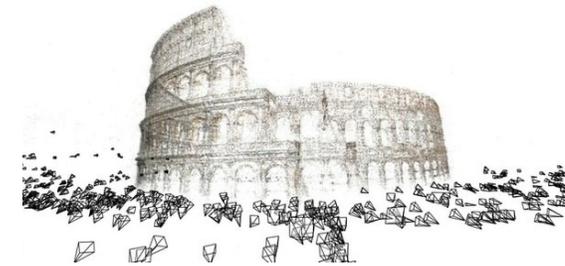
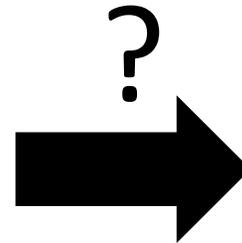
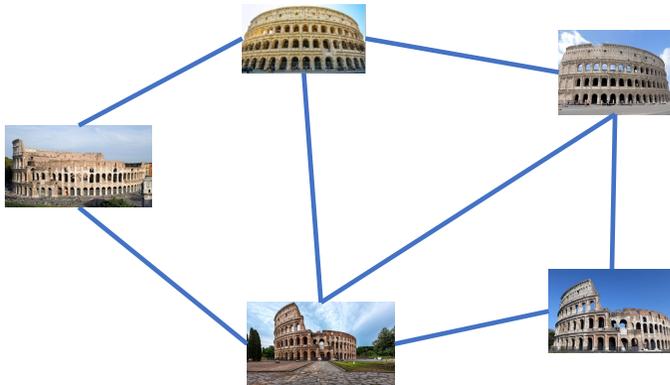


- Nodes = cameras/images
- Edges = two-view relations

Levi & Werman. *The viewing graph*. CVPR 2003.

# Introduction

For which graphs do we have a **well-posed** problem?



- ✅ A graph is called **solvable** if and only if the available two-view relationships **uniquely** (up to a single transformation) determine the cameras  $\rightarrow$  *unique solution*
- ❌ Otherwise it is called **non solvable**  $\rightarrow$  *multiple (infinitely many) solutions*

# Introduction

Here we focus on **solvability** only (*we do not address reconstruction*).

|                | Calibrated   | Uncalibrated   |
|----------------|--|--|
| Solvability    |  Arrigoni & Fusiello. <i>Bearing-based network localizability: a unifying view</i> . IEEE TPAMI (2019). |  Levi & Werman. <i>The viewing graph</i> . CVPR 2003.<br> Rudi, Pizzoli & Pirri. <i>Linear solvability in the viewing graph</i> . ACCV 2011.<br> Trager, Osserman, & Ponce. <i>On the solvability of viewing graphs</i> . ECCV 2018.<br> Arrigoni, Fusiello, Ricci & Pajdla. <i>Viewing graph solvability via cycle consistency</i> . ICCV (2021). |
| Reconstruction |  Ozyesil, Voroninski, Basri & Singer. <i>A survey of structure from motion</i> . Acta Numerica (2017).  |  Kasten, Geifman, Galun & Basri. <i>GPSfM: global projective SfM using algebraic constraints on multi-view fundamental matrices</i> . CVPR (2019)   |

It is important to check solvability **before running SfM**:

✓ If the graph is solvable, the SfM problem is well-posed.

✗ If the graph is not solvable, the problem is ill-posed: no method will return a useful solution.

# Outline

- Introduction

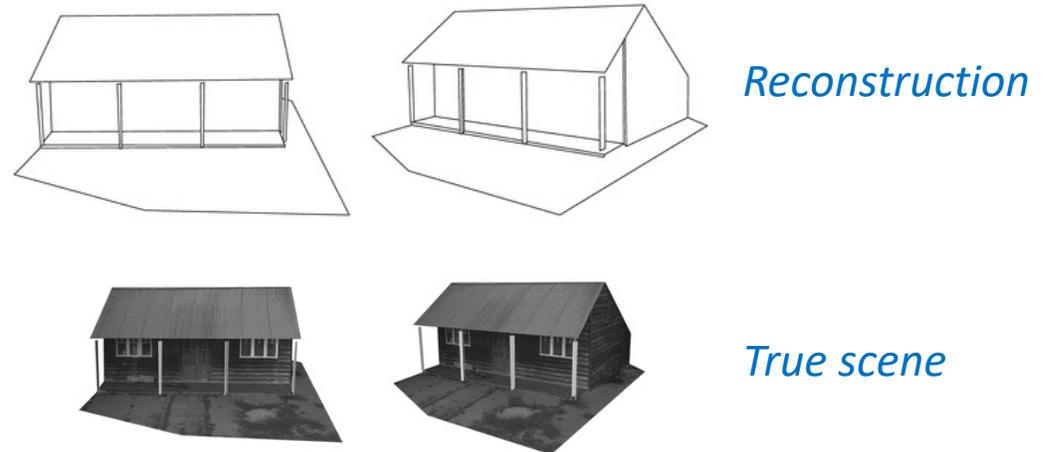
- **Calibrated Case** ----->

- Calibration matrix is required in advance
- Reconstruction is **metric** (up to scale)

- Uncalibrated Case

- Calibrated vs Uncalibrated

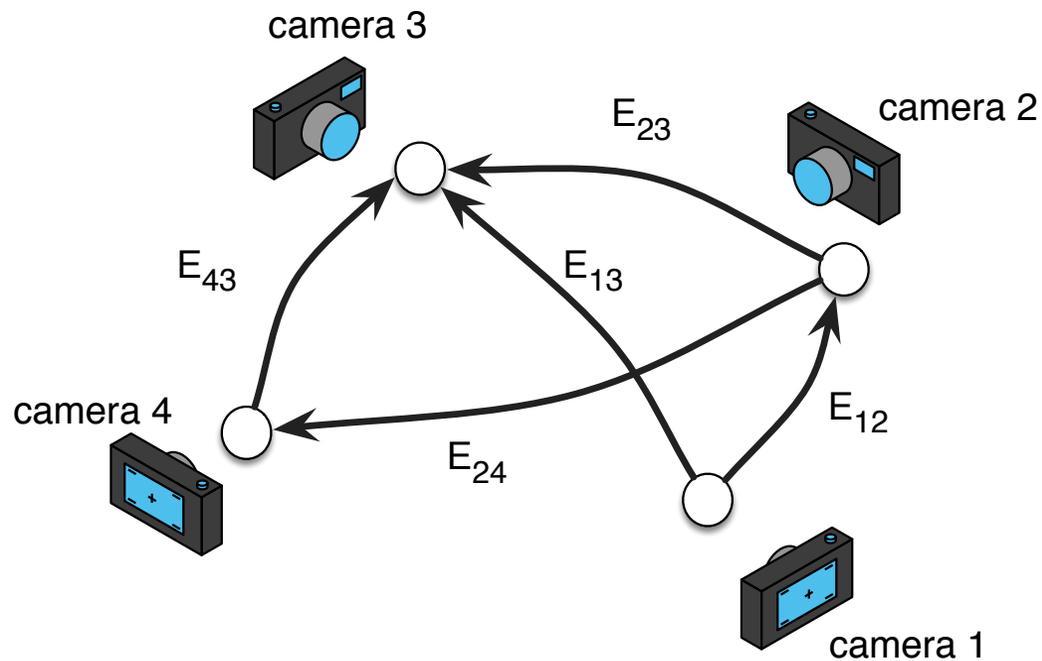
- Conclusion



# The Calibrated Case

## Problem Formulation

The **viewing graph** is a graph where vertices correspond to cameras and edges represent essential matrices.



*Each essential matrix can be decomposed into:*

- *Relative rotation  $R_{ij}$*
- *Relative translation  $t_{ij}$  (known up to scale)*

# The Calibrated Case

## Problem Formulation

**Solvable graph**  $\Leftrightarrow$  two-view transformations uniquely (up to a *single* rotation, translation & scale) determine the camera poses.

- We consider a **noiseless**-case
- We split the problem into **rotation and translation**:

$$\begin{aligned}
 R_{ij} &= R_i R_j^\top \\
 \mathbf{t}_{ij} &= -R_i R_j^\top \mathbf{t}_j + \mathbf{t}_i
 \end{aligned}
 \Leftrightarrow
 \begin{aligned}
 R_{ij} &= R_i R_j^\top \\
 \underbrace{-R_i^\top \mathbf{t}_{ij}}_{\mathbf{z}_{ij}} &= \underbrace{-R_i^\top \mathbf{t}_i}_{\mathbf{x}_i} + \underbrace{R_j^\top \mathbf{t}_j}_{-\mathbf{x}_j}
 \end{aligned}
 \xrightarrow{\text{Consistency constraint between relative and absolute poses}}$$

Relative displacement
Centre of camera i
Centre of camera j

# The Calibrated Case

## Problem Formulation

**Solvable graph**  $\Leftrightarrow$  two-view transformations uniquely (up to a *single* rotation, translation & scale) determine the camera poses.

- We consider a **noiseless**-case
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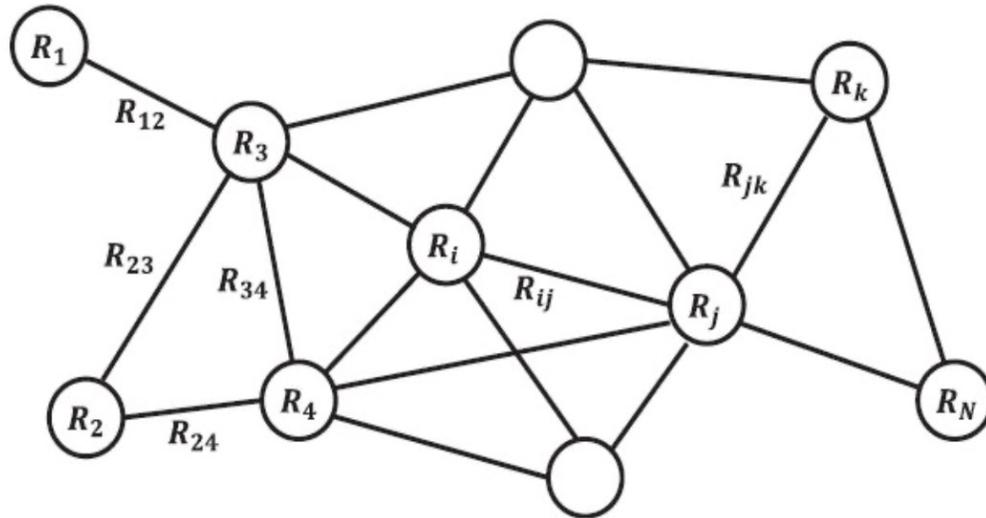
$$\begin{aligned} R_{ij} &= R_i R_j^\top \\ \mathbf{t}_{ij} &= -R_i R_j^\top \mathbf{t}_j + \mathbf{t}_i \end{aligned} \iff \begin{aligned} R_{ij} &= R_i R_j^\top \\ \underbrace{-R_i^\top \mathbf{t}_{ij}}_{\mathbf{z}_{ij}} &= \underbrace{-R_i^\top \mathbf{t}_i}_{\mathbf{x}_i} + \underbrace{R_j^\top \mathbf{t}_j}_{-\mathbf{x}_j} \end{aligned} \quad \text{-----> Consistency constraint between relative and absolute poses}$$

👉 The magnitude of relative translations are **unknown**:  $\|\mathbf{t}_{ij}\| = \|\mathbf{z}_{ij}\| = ?$

# The Calibrated Case

## Rotations

In which cases can we uniquely (up to a global rotation) recover camera rotations starting from relative rotations? 🤔



Given a **spanning tree**, a solution can be found by setting the root to the identity and propagating the consistency constraint:

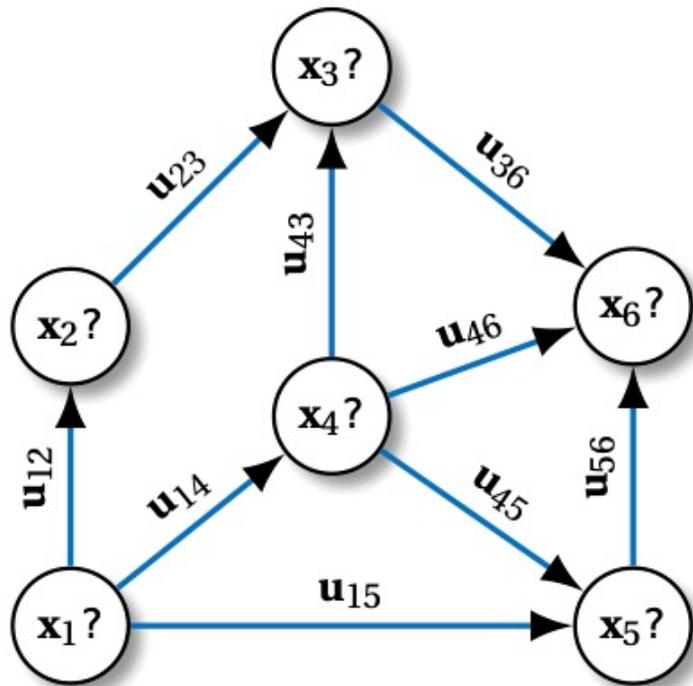
$$R_i = R_{ij}R_j \Leftrightarrow R_{ij} = R_iR_j^T$$

Solvability for rotations  $\Leftrightarrow$  **connected** viewing graph

# The Calibrated Case

## Translations

In which cases can we uniquely (up to translation & scale) recover camera positions from pairwise directions? 🤔



- **Nodes** = unknown locations
- **Edges** = known directions

$$\mathbf{u}_{ij} = \frac{\mathbf{x}_i - \mathbf{x}_j}{\|\mathbf{x}_i - \mathbf{x}_j\|} \iff \mathbf{u}_{ij} \times (\mathbf{x}_i - \mathbf{x}_j) = 0$$

*A solution can be found from the direction constraint, which is a **linear** equation!*

# The Calibrated Case

## Translations

**Theorem.** A graph is solvable if and only if  $\text{rank}(S)=3n-4$

Localization  
Equation:  $Sx=0$

Translation &  
scale ambiguity

✓ If the viewing graph is **solvable**, then the problem is well-posed.

✗ Otherwise, the problem is ill-posed: the **largest solvable component** has to be extracted  $\Leftrightarrow$  clustering rows in the null-space of  $S$

▣ F. Arrigoni, A. Fusiello. *Bearing-based network localizability: a unifying view*. IEEE TPAMI (2019).

▣ W. Whiteley. *Matroids from Discrete Geometry*. American Mathematical Society (1997)

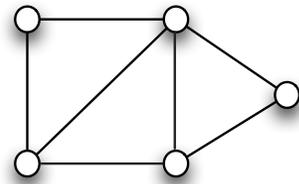
▣ R. Kennedy, K. Daniilidis, O. Naroditsky, C. J. Taylor. *Identifying maximal rigid components in bearing-based localization*. IROS (2012)

# The Calibrated Case

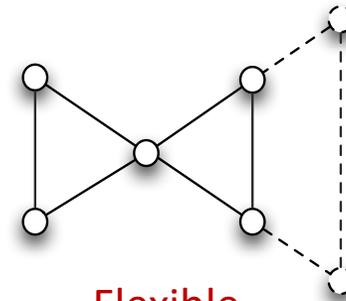
## Translations

Solvability for translations  $\Leftrightarrow$  **parallel rigid** viewing graph

**Definition.** A graph is **parallel rigid** when all the configurations with parallel edges differ by translation and scale. Otherwise it is called **flexible**.



Parallel rigid



Flexible

*This is a well studied task!*



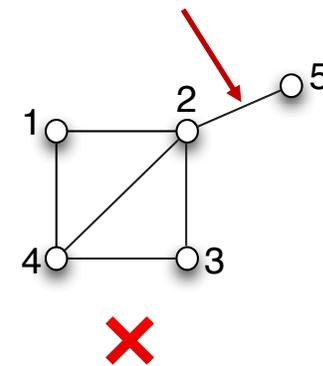
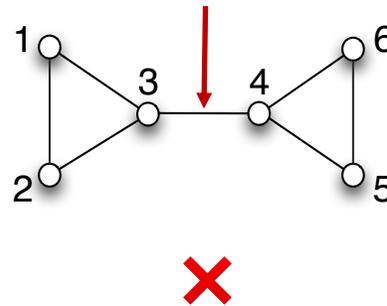
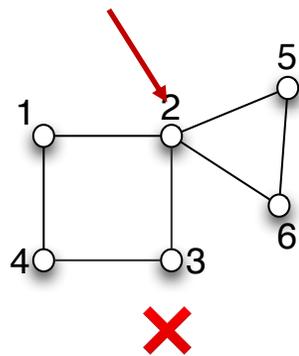
▣ O. Ozyesil, A. Singer. *Robust camera location estimation by convex programming*. CVPR (2015).

# The Calibrated Case

## Translations

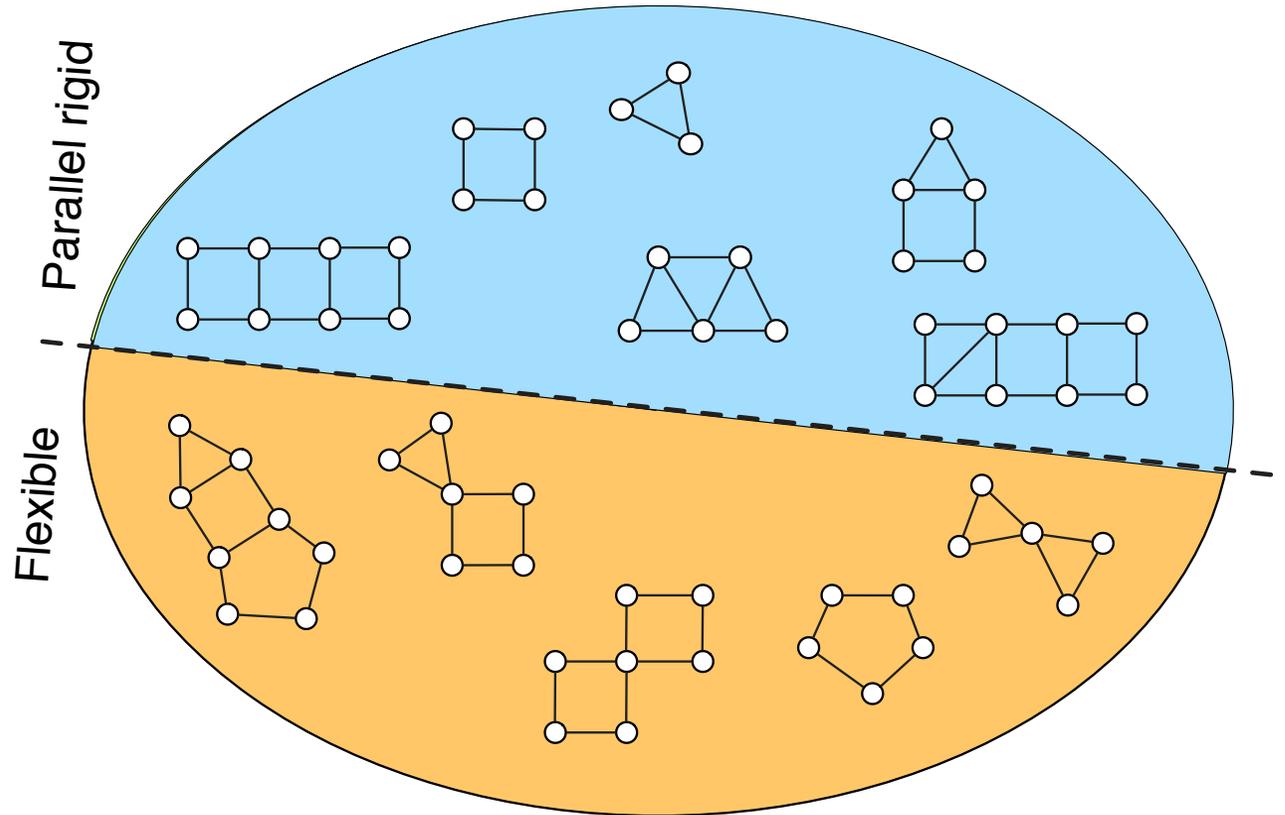
A parallel-rigid graph must satisfy the following **necessary conditions**:

- it has at least  $(3n-4)/2$  edges
- It is **bridgeless** (i.e., it remains connected after removing any edge).
- It is **biconnected** (i.e. it does not have **articulation points** meaning that it remains connected after removing any node).



# The Calibrated Case

## Examples



- A single cycle of **length 3 or 4** is parallel rigid, whereas longer cycles are flexible
- Union of rigid graphs with a common edge is also rigid  $\Rightarrow$  **sufficient conditions**

# The Calibrated Case

## Examples

| Dataset           | nodes | % edges | rigid | articulation | bridges |
|-------------------|-------|---------|-------|--------------|---------|
| Arts Quad         | 5530  | 2       | X     | 30           | 10      |
| Piccadilly        | 2508  | 10      | X     | 59           | 62      |
| Roman Forum       | 1134  | 11      | X     | 28           | 28      |
| Union Square      | 930   | 6       | X     | 60           | 68      |
| Vienna Cathedral  | 918   | 25      | X     | 19           | 20      |
| Alamo             | 627   | 50      | X     | 17           | 19      |
| Notre Dame        | 553   | 68      | ✓     | –            | –       |
| Tower of London   | 508   | 19      | X     | 19           | 19      |
| Montreal N. Dame  | 474   | 47      | X     | 7            | 7       |
| Yorkminster       | 458   | 26      | X     | 9            | 10      |
| Madrid Metropolis | 394   | 31      | X     | 17           | 15      |
| NYC Library       | 376   | 29      | X     | 17           | 18      |
| Piazza del Popolo | 354   | 40      | X     | 8            | 9       |
| Ellis Island      | 247   | 67      | X     | 6            | 7       |

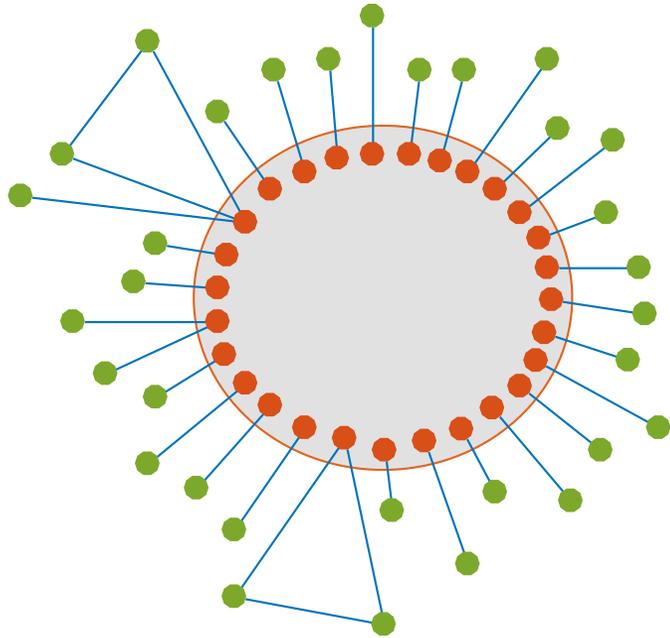
Cornell ArtsQuad <http://vision.soic.indiana.edu/projects/disco/>

1DSfM datasets <http://www.cs.cornell.edu/projects/1dsfm/>

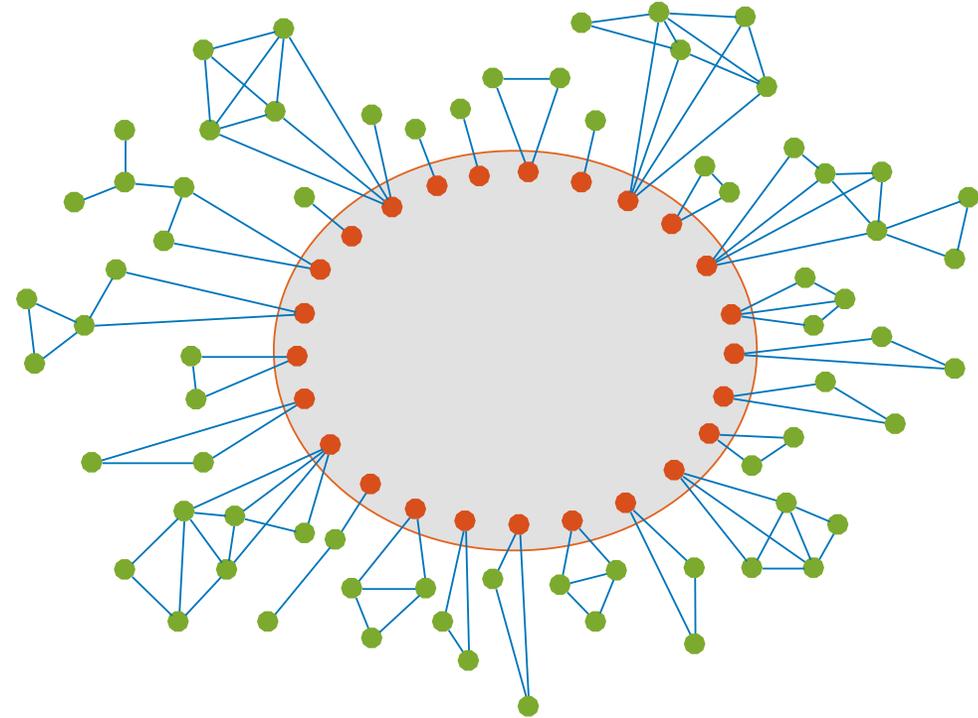
# The Calibrated Case

## Examples

**Simplified representation:** edges outside the largest rigid component are drawn.



Roman Forum



Arts Quad

# The Calibrated Case

## Summary

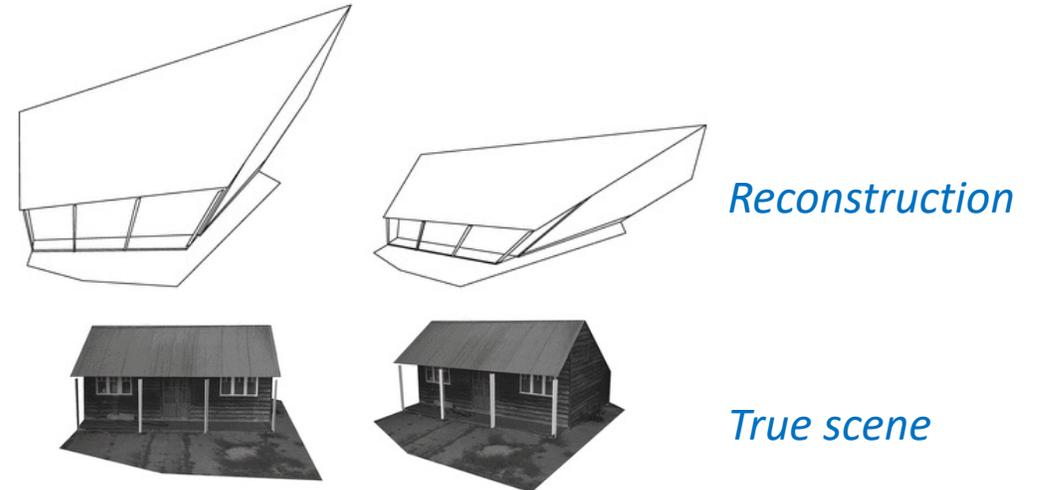
Solvability for rotations  $\Leftrightarrow$  **connected** viewing graph  
Solvability for translations  $\Leftrightarrow$  **parallel rigid** viewing graph

- Parallel rigidity can be tested from the rank of a **linear system**.
- Maximal components can be extracted from the **null-space** of such a system.
- **Large-scale** datasets can be processed. 😊

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- Introduction
- Calibrated Case
- **Uncalibrated Case** →
- Calibrated vs Uncalibrated
- Conclusion

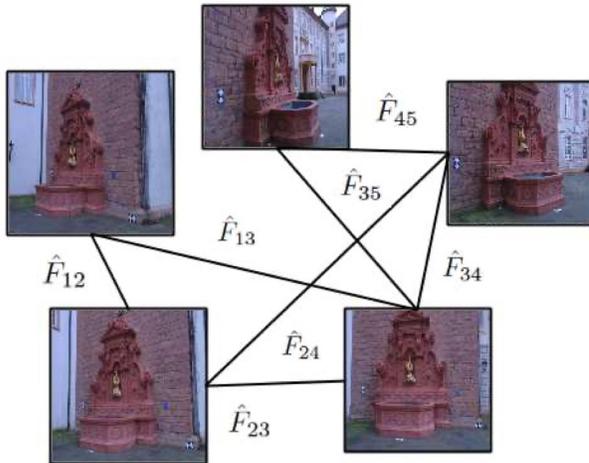
- No assumptions
- Reconstruction is **projective**



# The Uncalibrated Case

## Problem Formulation

The **viewing graph** is a graph where vertices correspond to cameras and edges represent fundamental matrices.



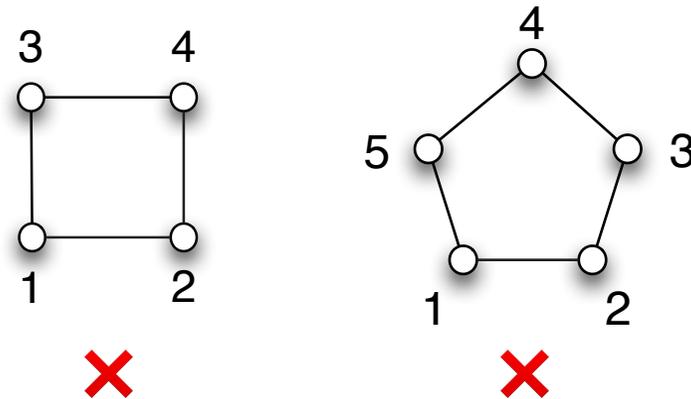
- Solvability depends on the **graph** and **camera centres** only.
- It can be reduced to a property of the graph only if we assume **generic** centres.

**Solvable graph**  $\Leftrightarrow$  it uniquely (up to a *single* projective transformation) determines a projective configuration of cameras.

# The Uncalibrated Case

## Necessary Conditions

- A solvable graph has at least  $(11n-15)/7$  edges.
- In a solvable graph, all the vertices have degree at least two and no two adjacent vertices have degree two (if  $n > 3$ ).



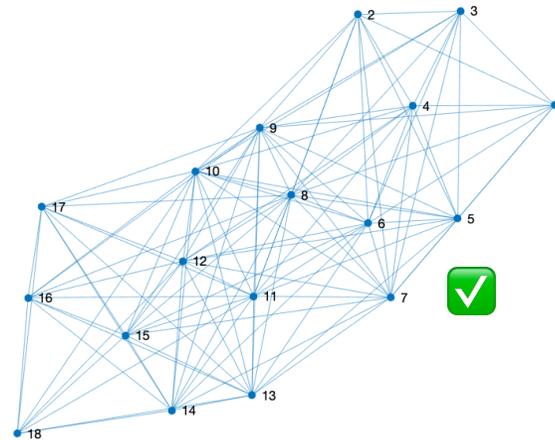
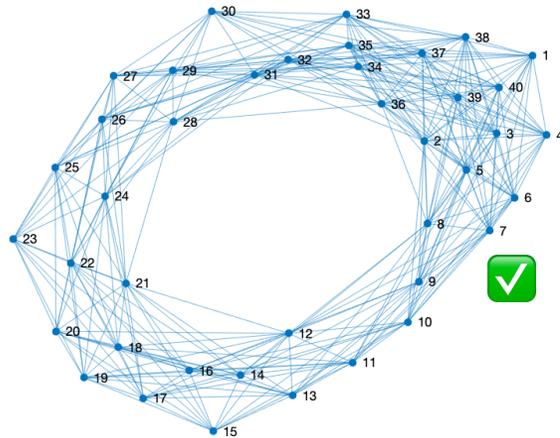
■ M. Trager, B. Osserman, and J. Ponce. *On the solvability of viewing graphs*. ECCV 2018.

■ N. Levi and M. Werman. *The viewing graph*. CVPR 2003

# The Uncalibrated Case

## Sufficient Conditions

- **Triangulated** graphs are solvable
- **Constructive** approaches are also available



▣ M. Trager, M. Hebert, and J. Ponce. *The joint image hand-book*. ICCV 2015.

▣ A. Rudi, M. Pizzoli, and F. Pirri. *Linear solvability in the viewing graph*. ACCV 2011.

# The Uncalibrated Case

## Algebraic Characterization

**Idea:** characterize the set of projective transformations that represent all possible ambiguities of the problem.

First, let us identify the family of transformations that leave a **single camera** fixed.

**Proposition.** Let  $P$  be a camera with centre  $c$ . All the solutions to  $PG = aP$  for  $G \in GL(4, \mathbb{R})$  and  $a \in \mathbb{R}_{\neq 0}$  are given by  $G = aI_4 + \mathbf{c}\mathbf{v}^T$   $\forall a \in \mathbb{R}_{\neq 0}, \mathbf{v} \in \mathbb{R}^4$

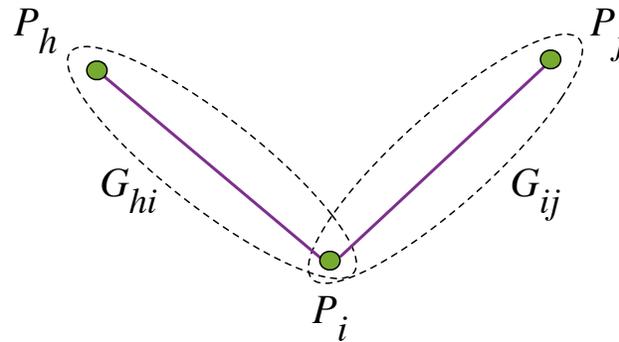
■ M. Trager, B. Osserman, and J. Ponce. *On the solvability of viewing graphs*. ECCV 2018.

# The Uncalibrated Case

## Algebraic Characterization

What happens when we have **multiple cameras**, represented as a viewing graph? 🤔

Let us assign an unknown projective transformation  $G_{ij}$  to every edge, and let us consider two edges  $(h, i)$  and  $(i, j)$  with a common vertex  $i$ .



### Compatibility Condition

$$G_{hi}G_{ij}^{-1} = a_{hij}I_4 + \mathbf{c}_i\mathbf{v}_{hij}^T$$

$G_{hij} \in GL(4)$  is unknown

$a_{hij} \in \mathbb{R}_{\neq 0}$  and  $\mathbf{v}_{hij} \in \mathbb{R}^4$  are unknown

$\mathbf{c}_i \in \mathbb{R}^4$  is known (camera center)

$$\text{Solvable graph} \Leftrightarrow G_{ij} = s_{ij} H$$

Single projective transformation

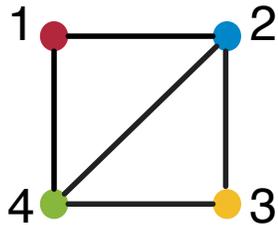
■ M. Trager, B. Osserman, and J. Ponce. *On the solvability of viewing graphs*. ECCV 2018.

# The Uncalibrated Case

## Algebraic Characterization

### Input Graph

$$G_{hi}G_{ij}^{-1} = a_{hij}I_4 + \mathbf{c}_i\mathbf{v}_{hij}^T$$



- **Polynomial** system of equations with many unknowns

$G_{hi} \in GL(4)$  is unknown

$a_{hij} \in \mathbb{R}_{\neq 0}$  and  $\mathbf{v}_{hij} \in \mathbb{R}^4$  are unknown

$\mathbf{c}_i \in \mathbb{R}^4$  is known (camera center)

$(h, i)$  and  $(i, j)$  are adjacent edges

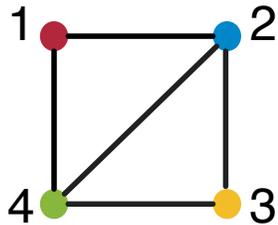
▣ M. Trager, B. Osserman, and J. Ponce. *On the solvability of viewing graphs*. ECCV 2018.

# The Uncalibrated Case

## Reduced Formulation

### Input Graph

$$G_{hi}G_{ij}^{-1} = a_{hij}I_4 + \mathbf{c}_i\mathbf{v}_{hij}^T$$



- **Polynomial** system of equations with many unknowns

$G_{hi} \in GL(4)$  is unknown

$a_{hij} \in \mathbb{R}_{\neq 0}$  and  $\mathbf{v}_{hij} \in \mathbb{R}^4$  are unknown

$\mathbf{c}_i \in \mathbb{R}^4$  is known (camera center)

$(h, i)$  and  $(i, j)$  are adjacent edges

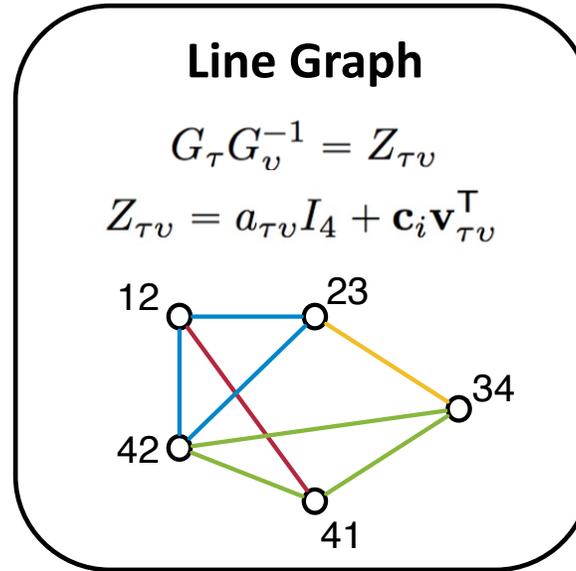
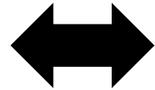
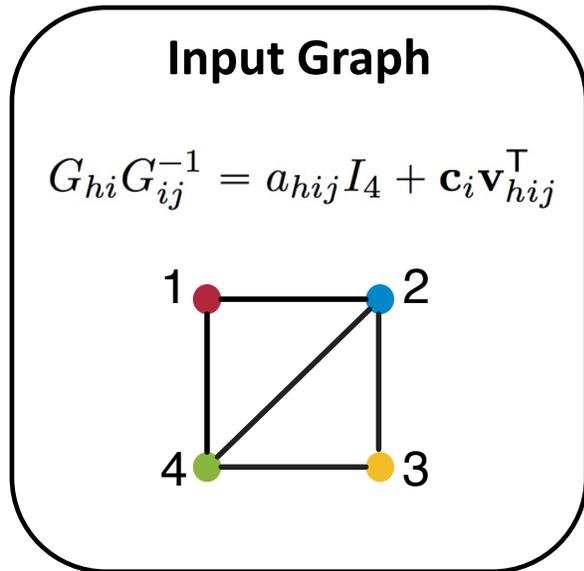
▀ M. Trager, B. Osserman, and J. Ponce. *On the solvability of viewing graphs*. ECCV 2018.

- It is possible **eliminate variables** 😊

▀ Arrigoni, Fusiello, Ricci & Pajdla. *Viewing graph solvability via cycle consistency*. ICCV (2021).

# The Uncalibrated Case

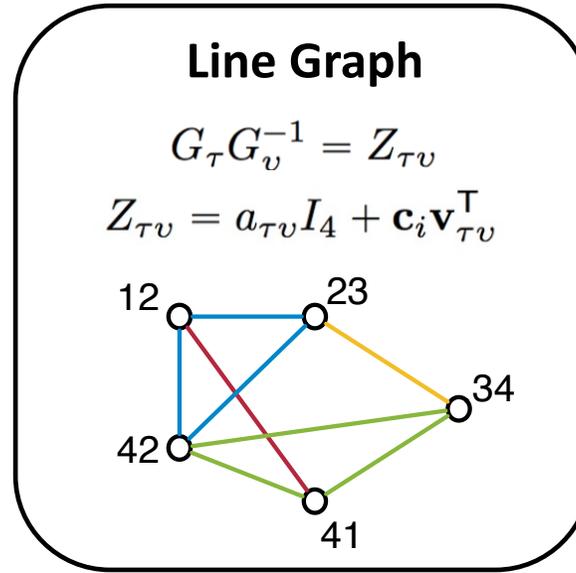
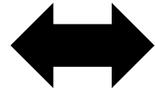
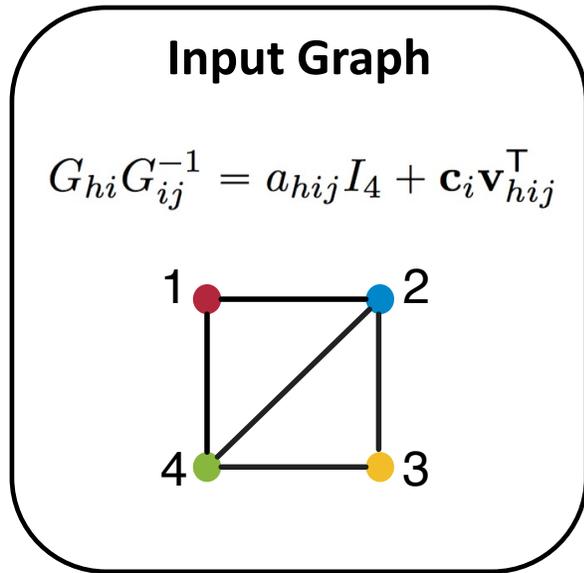
## Reduced Formulation



- Each node is an edge in the input graph;
- Two nodes are linked if the corresponding edges are adjacent in the input graph.
- There is one equation for each edge in the line graph.

# The Uncalibrated Case

## Reduced Formulation



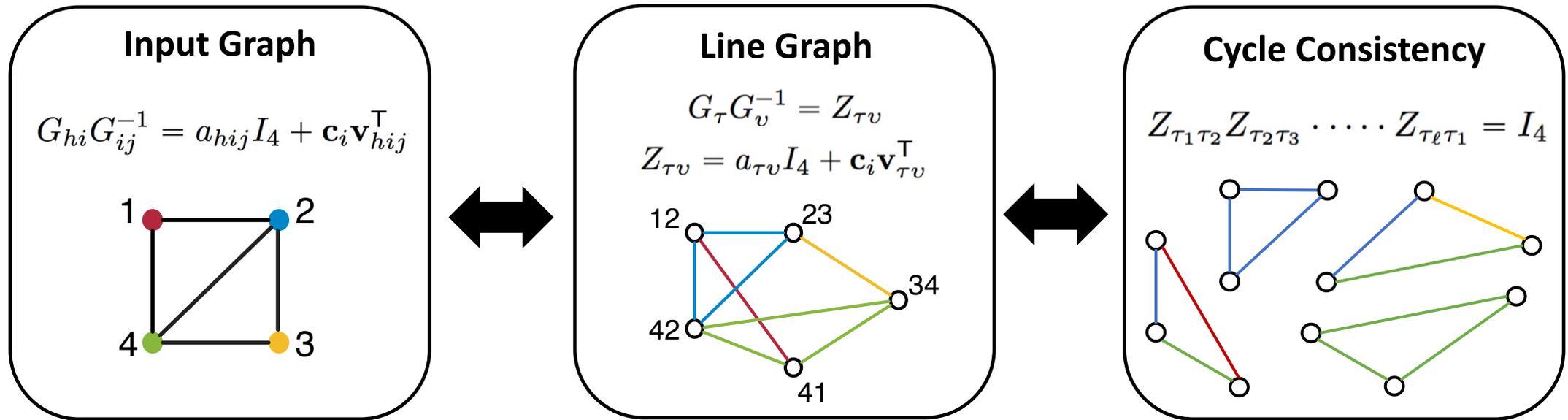
- Each node is an edge in the input graph;
- Two nodes are linked if the corresponding edges are adjacent in the input graph.
- There is one equation for each edge in the line graph.

How can we eliminate the **G** variables? 🤔

**Idea:**  $Z_{12,23} \cdot Z_{23,42} \cdot Z_{42,12} = G_{12} \underbrace{G_{23}^{-1}G_{23}}_I \underbrace{G_{42}^{-1}G_{42}}_I G_{12}^{-1} = I$

# The Uncalibrated Case

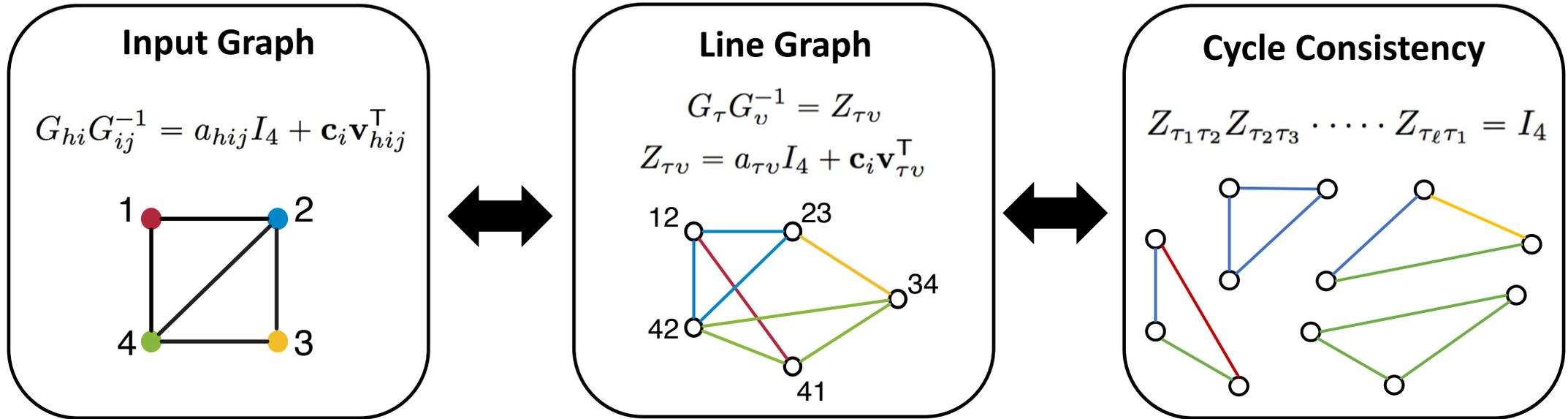
## Reduced Formulation



cycle consistency (on all cycles)  $\Leftrightarrow$  cycle consistency (on a basis)

# The Uncalibrated Case

## Reduced Formulation



The formulation can be simplified via a **change of variables**:  $\mathbf{u}_{\tau\nu} = \mathbf{v}_{\tau\nu}/\alpha_{\tau\nu}$   
 $\Rightarrow$  For a solvable graph, we have exactly 1 solution (no ambiguities)

# The Uncalibrated Case Algorithm

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**Algorithm 1** Viewing Graph Solvability

---

**Input:** undirected graph  $\mathcal{G} = (\mathcal{V}, \mathcal{E})$

**Output:** solvable or not solvable

1. randomly sample the camera centres
2. compute the line graph  $\mathcal{L}(\mathcal{G})$
3. compute a cycle consistency basis for  $\mathcal{L}(\mathcal{G})$
4. set up equations
5. compute the number  $s$  of real solutions

**if**  $s = 1$  **then**

solvable 

**else**

not solvable 

**end if**

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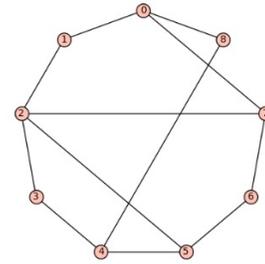
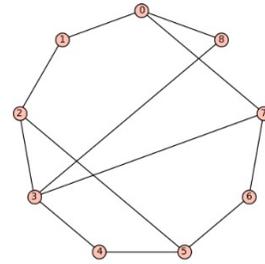
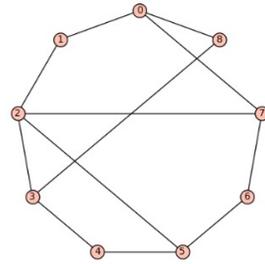
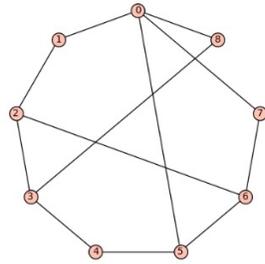
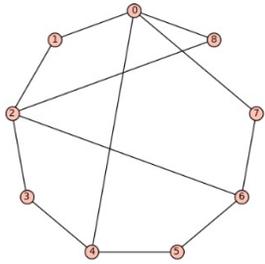
**Gröbner basis**  
(symbolic computation)

<https://github.com/federica-arrigoni/solvability>

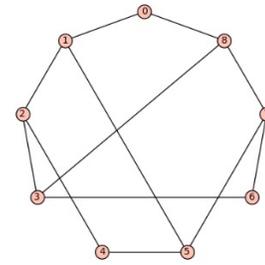
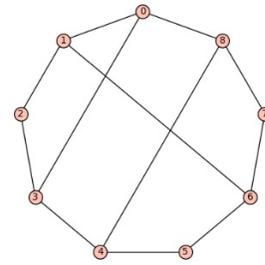
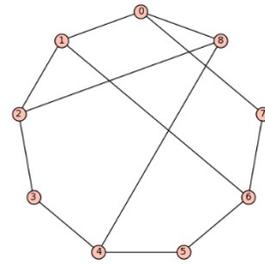
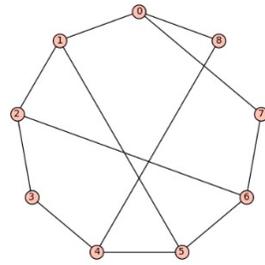
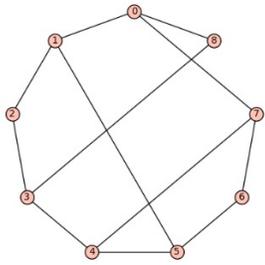
# The Uncalibrated Case

## Examples

### Minimal viewing graphs with 9 vertices



Solvable 



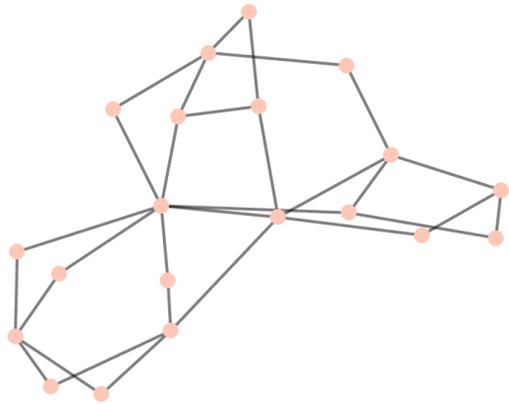
Not solvable 

# The Uncalibrated Case

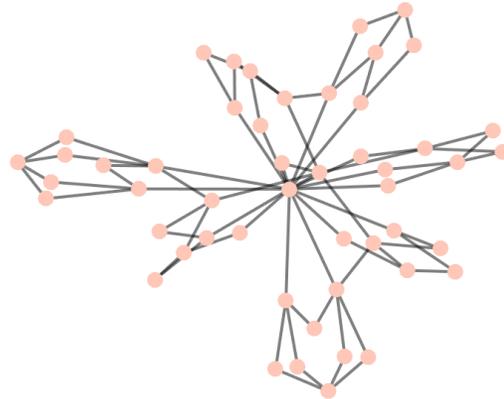
## Examples

### Execution times on minimal graphs

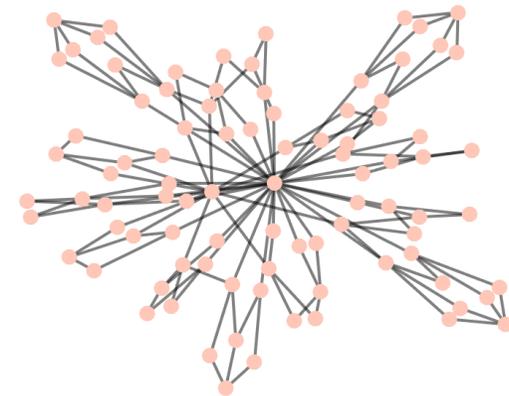
|       |       |     |      |       |        |        |     |       |       |
|-------|-------|-----|------|-------|--------|--------|-----|-------|-------|
| Nodes | 10    | 20  | 30   | 40    | 50     | 60     | 70  | 80    | 90    |
| Time  | 1.6 s | 9 s | 93 s | 3 min | 15 min | 35 min | 1 h | ≈ 2 h | > 4 h |



Solvable graph with 20 nodes



Solvable graph with 50 nodes

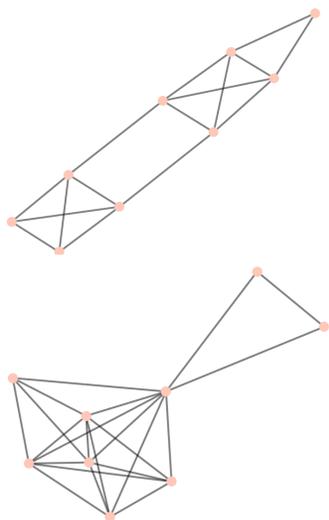


Solvable graph with 90 nodes

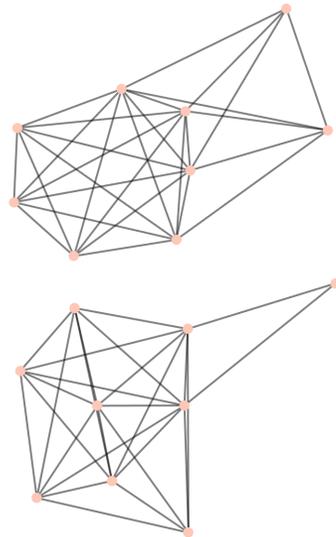
# The Uncalibrated Case

## Examples

Subgraphs with 9 nodes sampled from  
**real structure-from-motion viewgraphs**



Unsolvable



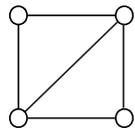
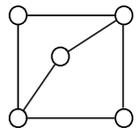
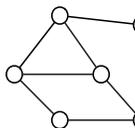
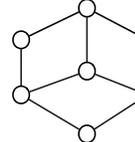
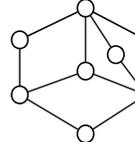
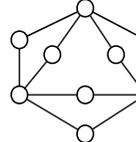
Solvable

| Data set            | Solvable |           |      | Unsolvable |           |      |
|---------------------|----------|-----------|------|------------|-----------|------|
|                     | by suff. | by Alg. 1 | Tot. | by nec.    | by Alg. 1 | Tot. |
| Alcatraz Courtyard  | 200      | 0         | 200  | 0          | 0         | 0    |
| Buddah Tooth        | 178      | 20        | 198  | 2          | 0         | 2    |
| Pumpkin             | 169      | 22        | 191  | 8          | 1         | 9    |
| Skansen Kronan      | 179      | 8         | 187  | 13         | 0         | 13   |
| Tsar Nikolai I      | 196      | 0         | 196  | 4          | 0         | 4    |
| Alamo               | 136      | 16        | 152  | 48         | 0         | 48   |
| Ellis Island        | 136      | 30        | 166  | 34         | 0         | 34   |
| Gendarmenmarkt      | 128      | 11        | 139  | 61         | 0         | 61   |
| Madrid Metropolis   | 88       | 28        | 116  | 84         | 0         | 84   |
| Montreal Notre Dame | 140      | 12        | 152  | 48         | 0         | 48   |
| Notre Dame          | 165      | 18        | 183  | 17         | 0         | 17   |
| NYC Library         | 110      | 19        | 129  | 71         | 0         | 71   |
| Piazza del Popolo   | 105      | 22        | 127  | 73         | 0         | 73   |
| Piccadilly          | 109      | 23        | 132  | 68         | 0         | 68   |
| Roman Forum         | 114      | 28        | 142  | 58         | 0         | 58   |
| Tower of London     | 123      | 18        | 141  | 59         | 0         | 59   |
| Trafalgar           | 86       | 16        | 102  | 98         | 0         | 98   |
| Union Square        | 74       | 19        | 93   | 107        | 0         | 107  |
| Vienna Cathedral    | 122      | 8         | 130  | 70         | 0         | 70   |
| Yorkminster         | 116      | 14        | 130  | 70         | 0         | 70   |
| Cornell Arts Quad   | 76       | 23        | 99   | 101        | 0         | 101  |

# The Uncalibrated Case

## Summary

- Thanks to cycle consistency, **less unknowns** are involved than previous work:

|                 |  |  |  |  |  |  |
|-----------------|---|--|---|---|---|---|
|                 | #Eq. #Var.  | #Eq. #Var.   | #Eq. #Var.  | #Eq. #Var.  | #Eq. #Var.  | #Eq. #Var.  |
| Our formulation | 64 36   | 64 40  | 112 63  | 112 67  | 192 100   | 208 109   |
| Trager et al.   | 128 120   | 144 141  | 224 198   | 240 219   | 352 286   | 384 312   |

- It is possible to classify **previously undecided** viewing graphs and extend solvability testing up to minimal graphs **with 90 nodes**.
- Larger/denser graphs can not be processed* 😞

# Outline

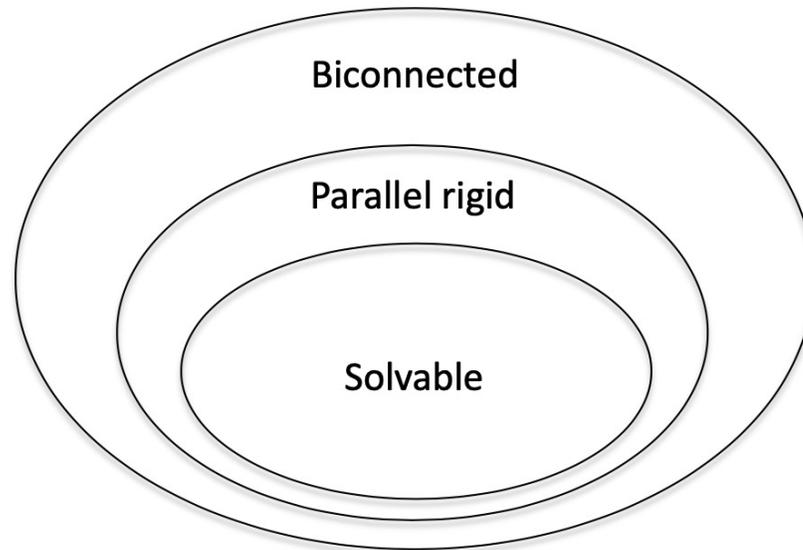
- Introduction
- Calibrated Case
- Uncalibrated Case
- **Calibrated vs Uncalibrated** 
- Conclusion



Uncalibrated solvability  $\Rightarrow$  calibrated solvability

# Calibrated vs Uncalibrated

**Proposition.** *A solvable (uncalibrated) graph is parallel rigid.*



*Expected result!* 😊💧

Well-posed with uncalibrated cameras  
⇒ well-posed with calibrated cameras

▣ Arrigoni, Fusiello, Rizzi, Ricci & Pajdla. *Revisiting viewing graph solvability: an effective approach based on cycle consistency*. TPAMI (2022).

# Calibrated vs Uncalibrated

**Proposition.** *A solvable (uncalibrated) graph is parallel rigid.*

**Proof [sketch].**

Parallel rigid graph  $\Leftrightarrow$  for any partition of the edges:  $\sum_{i=1}^k (3|\mathcal{V}_i| - 4) \geq 3n - 4$

Solvable graph  $\Rightarrow$  for any partition of the edges:  $\sum_{i=1}^k (11|\mathcal{V}_i| - 15) \geq 11n - 15$

*Only necessary condition!  
Unknown if the opposite holds*

▀ Arrigoni, Fusiello, Rizzi, Ricci & Pajdla. *Revisiting viewing graph solvability: an effective approach based on cycle consistency*. TPAMI (2022).

# Outline

- Introduction
- Calibrated Case
- Uncalibrated Case
- Calibrated vs Uncalibrated
- **Conclusion**

# Conclusion

|                | <b>Calibrated</b>          | <b>Uncalibrated</b> |
|----------------|----------------------------|---------------------|
| Formulation    | Linear system              | Polynomial system   |
| Datasets       | Large-scale                | Small-scale         |
| Interpretation | Connected + Parallel rigid | ?                   |
| Components     | Null-space computation     | ?                   |

↓  
“Solved”

↓  
Open issues

# References

- 📖 F. Arrigoni, A. Fusiello, R. Rizzi, E. Ricci & T. Pajdla. *Revisiting viewing graph solvability: an effective approach based on cycle consistency*. IEEE TPAMI (2022).
- 📖 F. Arrigoni, A. Fusiello, E. Ricci & T. Pajdla. *Viewing graph solvability via cycle consistency*. ICCV (2021). **Best paper honourable mention** 🏆
- 📖 F. Arrigoni & A. Fusiello. *Bearing-based network localizability: a unifying view*. IEEE TPAMI (2019).

*Thank you for your attention!*

# Viewing Graph Solvability in Structure from Motion

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